Psychological Review
1976, Vol. 83, No. 1, 37-64

The Cognitive Side of Probability Learning

W. K. Estes
Rockefeller University

Models proposed for probability learning largely represent performance rather than learning theory. Perhaps for this reason, quite different models have been required to provide accounts of data arising from different experimental paradigms. In the present approach, a common theoretical framework is sought in concepts of coding and organization in memory. Following the theoretical analysis, an observation-transfer paradigm is developed that permits the study of predictive behavior depending on categorical, as distinguished from episodic, memory. This paradigm yields evidence that probability learning and transfer derive from frequency learning. The individual categorizes events and forms representations in memory of relative frequencies of event categories. When the different cues in a multiple-cue, probability learning situation occur equally often, this process yields predictive behavior closely reflecting the probabilities that the alternative events associated with a cue will occur when the cue is present. But when cue frequencies are unequal, the categorical memory model implies (correctly) that predictive behavior may be grossly out of line with actual probabilities. In general, depending on task requirements, predictive responses are either direct reflections of relative frequency judgments or are governed by strategies involving an additional level of encoding of event categories.

Probability learning has been somewhat eclipsed in the literature of cognitive psychology by an increasing preoccupation with psycholinguistics and the semantic aspects of memory. Nonetheless, it should be recognized that we are scarcely in a position to close the chapter of research on this aspect of human learning. First, the reasons, both theoretical and practical, that were responsible for the wave of interest in probability learning during the 1950s have by no means evaporated. Rather, it continues to be apparent that probability learning constitutes a major interface between cognitive psychology and the practical world. Second, major problems of methodology and interpretation remain unsolved. And third, as I propose to show in the following pages, new findings are emerging that may bring the study of probability learning closer to the mainstream of research on human memory and information processing.

Some Problems in Need of a Theory

A theme expressed earlier by, for example, Cohen (1964) and Restle (1961) is the keynote of a new exploration by Kahneman and Tversky (1972) of the fallibility of human probability judgments: "Subjective probabilities play an important role in our lives. The decisions we make, the conclusions we reach, and the explanations we offer are usually based on our judgments of the likelihood of uncertain events" (p. 430).

Among the specific areas where probability learning can be expected to play a central role are economics, clinical judgment and medical diagnosis, and the control of human behavior by reinforcement. In the area of economics, besides the techniques already available for assessing opinions and beliefs after the fact, a body of theory is required that will enable prediction of changes in people's beliefs and expectations in response to

This paper was presented, in substance, as an invited address at the May, 1974 meetings of the Midwestern Psychological Association.

Research reported herein was supported by Public Health Service Grants MH16100 and MH23878 from the National Institute of Mental Health.

I am indebted to Edith Skaar for assistance with every aspect of the experimental work and data processing.

Requests for reprints should be sent to W. K. Estes, The Rockefeller University, New York, New York 10021.

fluctuations in economic variables. In the medical area, it has been shown in both experimental (Estes, 1972a) and clinical (Goldberg, 1970) settings that human judges whose tasks require them to make decisions on the basis of their knowledge of probabilities perform with far from maximal efficiency. In fact, they are often outdone by models which consistently use the same decision strategies that are manifested, though less uniformly, in their own behavior. Thus, a major question remains to be answered by research and theory: Where do the judges fall short—in processing information regarding probabilities of uncertain events or in making choices based on states of information?

With regard to the control of human behavior by reinforcement, the old picture of the human learner being shaped relentlessly by the effects of rewards and punishments via their strengthening and weakening influences on stimulus–response connections or habit strengths has given way to a view of the human organism as an information processor and decision maker using, rather than being driven by, informative feedback from the consequences of his actions. The individual is seen as actively sampling the alternative courses of action available in a choice situation, generating expectations about the probable consequences of the actions based on his past experience, and tending to select the responses yielding the higher expectations of success (Atkinson & Wickens, 1971; Estes, 1969, 1972b; Greeno, 1968). However, in the research that is needed to subject this newer conception to rigorous test, expectations and decisions can only be inferred from performance. Once again, adequate models are needed to provide a framework within which we can effectively study the hypothetical processes and mechanisms.

Alternative Models for Probability Learning

The expenditure over several decades of a great deal of effort directed toward the development of such models has led to an extremely mixed picture. Even if we limit attention to models that have evolved within the framework of learning theory, three main types can be identified: (a) incremental learning models, (b) all-or-none coding models, and (c) hypothesis-testing models. In the first category are the familiar linear models of stimulus-sampling theory (Bush & Mosteller, 1955; Estes & Straughan, 1954); in the second, the pattern model (Estes, 1959) and schema models (Restle, 1961). In the third class are Bayesian approaches, which, in effect, treat the learner as an intuitive statistician (Shuford, 1964), and more psychologically oriented models such as that of Castellan and Edgell (1973).

A curious outcome of numerous applications of these three types of models to data is that each has produced some striking successes. The linear, or stimulus-sampling, models have provided quite satisfactory descriptions of the course and terminal level of probability learning for a wide range of experiments on predictive behavior over limited numbers of trials (typically 1 to 300) and a close account of numerous detailed properties of data in some situations (Estes, 1972c; Friedman et al., 1964; Suppes & Atkinson, 1960). Discrepancies between models and data begin to appear as one deviates from the restrictive boundary conditions and attempts to deal with long sequences of trials, with many alternative choices, or with asymmetric payoffs for correct and incorrect predictions.

Predictions from the pattern model agree with those of the linear model regarding overall learning curves in most situations, but they differ with respect to variances and sequential statistics. In two-choice situations with noncontingent event probabilities, the pattern model has proven superior to the linear model in predicting detailed sequential properties of data and has in fact been notably successful in a few instances (Yellott, 1969).

Hypothesis-testing models have dealt largely with asymptotic behavior. Even the best developed model of this type, that of Castellan and Edgell (1973), does not attempt to account for the course of learning but simply assumes that the learner somehow develops subjective probabilities at least roughly in accord with objective probabilities. Hypothesis-testing models have in a number of cases provided excellent accounts of asymptotic performance even in quite complex situations (Castellan & Edgell, 1973; Friedman, Rollins, & Padilla, 1968), but they have not addressed the details of learning or sequential properties of performance.

Although none of these models comes close to providing a satisfactory account of the broad range of probability learning data, the instances of accurate predictions for substantial bodies of data in particular cases can scarcely be attributed to chance. Evidently, the different models are capturing different aspects of a complex process, some aspects being more prominent in some situations. One would like to replace the collection of locally successful models with one general theory, but this objective may not be within our capabilities. A more feasible immediate goal may be to try to understand why different models are required to deal with different situations.

On the Separation of Information and Decision Processes

Progress toward this goal may depend on better analyses both of learning and of performance. However, closely intermeshed these may be in the individual's behavior, we can at least conceptually separate questions concerning the nature of the information an individual acquires about environmental probabilities and the way in which he generates choices on the basis of this acquired information. In the present study I shall concentrate on questions of the former type for two reasons: First, in the literature of probability learning, the question of exactly *what* is learned has been relatively neglected; second, in the current literature of cognitive psychology, there is much that is new in both methodology and theories of memory and information processing that might be, but so far has not been, applied to the interpretation of probability learning.

Extant theories of probability learning are primarily, though to varying extents, performance rather than learning models. They have been concerned mostly with what the individual achieves in a choice situation rather than with the basis for achievement. The so-called "learning models" have been primarily concerned with the rules for changes in performance; the hypothesis-testing models, with the way in which the individual performs when his memory is characterized by various possible states of knowledge. Thus, the basic assumptions of the linear and stimulus-sampling models prescribe how the probabilities of various predictive responses by the learner change as a function of outcomes. In the pattern models it is presumed that the individual succeeds in encoding recurring patterns of stimulation as units; the assumptions of the pattern models prescribe the probabilities with which the individual attends to different types of patterns and the predictive responses he makes when these patterns occur. In hypothesis-testing models, the formal assumptions specify the probabilities with which the subject selects various stimulus dimensions or cues as relevant to a choice problem; they also specify the decision mechanism as a process of basing choices on the subjective probabilities of various outcomes occurring when these relevant cues are attended to (Castellan & Edgell, 1973).

None of the models of any of the three classes comes to grips with the problem of how information on environmental probabilities is represented in the memory system. Thus the possibility arises that attention to this neglected problem might prove fruitful in a number of respects. Ideas drawn from current theories of memory and information processing may help to organize both the phenomena and the theories and may provide clues to why such different models seem required under different circumstances.

The Relevance of Concepts of Memory

In the experimental analysis of probability learning, research seems often to have proceeded from one study to the next with little attention to reformulating the problems from time to time in the light of theoretical advances in related areas. It may be instructive to step back for a moment and attempt to place the kind of behavior studied in these experiments in the broader context of learning and memory in relation to environmental uncertainties.

In general, we generate predictions by recognizing a new situation as one of a class of situations to which some rule applies. The rules with which we have to deal can be ordered in terms of complexity. For the normal adult, the rules often take the form of laws, principles, or formulas gained from scientific or technical training. For the most part, predictive behavior based on these formal rules is beyond our present capacity for experimental or theoretical analysis; perhaps the one small step beyond sheer description is to be found in the studies of sequence learning (e.g., Myers, in press; Restle & Brown, 1970; Wolin, Weichel, Terebinski, & Hansford, 1965).

But the predictive behavior of animals and young children, and even much of that observed in adults, is based not on formal rules but only on the experience an individual gains from his observation of recurrences of a given type of situation, together with his faith in the uniformity of nature. In everyday life, as well as in science, we always tend to assume that the repetition of the same combination of circumstances will lead to the same outcome. Concepts of causality and determinism express an idealization of this rule of thumb as an abstract principle, representing a limiting case of our experience rather than a result of direct observation. What we do observe is that, generally, the more nearly circumstances are reinstated, the greater is the likelihood of the same outcome.

When repeated occurrences of apparently the same combination of circumstances yield different outcomes, we form expectations on the basis of relative frequencies. The most frequent outcome in the past is assumed, other things being equal, to be the most likely in the future. The study of the way these expectations develop in simplified situations that provide no other sources of information is the study of probability learning as this concept has evolved in the research literature (Björkman, 1966).

In terms of current cognitive theory, what processes should we expect to play important roles in this type of learning? A logical analysis of the problem of prediction brings out aspects relative to both sides of the distinction between episodic and semantic memory, a distinction which has become increasingly prominent in current research and theory. As introduced by Tulving (1972), *episodic memory* refers to recall or recognition of events in context. Restle's schema theory might be regarded as a direct extension of the concept of episodic memory. Restle (1961) assumes that the full pattern of events occurring on each trial of an experiment gives rise to a schema that is stored as a unit in the memory system; the basis for an individual's predictive behavior in new situations is a comparison of the new pattern of stimulation with the various schemata stored in memory, with the expectation that the outcome of the present situation will be the same as the outcome of the most similar situation represented in the memory store. The pattern model (Estes, 1959) can be given a similar interpretation.

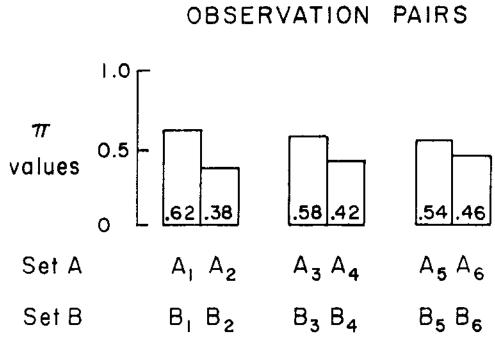
The counterpart to episodic memory in Tulving's classification is *semantic memory,* referring to the long-term representation of concepts and relations between concepts. However, it should be noted that concepts can be defined in terms of classes of events and need not be linguistic in character. Thus, I suggest that a better classification for our purposes would be *episodic* versus *categorical* memory. In the case of probability learning, categorical memory refers to the representation of relative frequencies of classes, or categories, in memory. These representations may be tapped directly in experiments on verbal discrimination learning (Ekstrand, Wallace, & Underwood, 1966) and relative frequency judgments (Hintzman, 1969). It seems pertinent to inquire whether these representations may not also play an important role in predictive behavior and probability estimates.

An Experimental Design for the Analysis of Probability Learning Based on Categorical Memory

If analysis of the probability learning situation in terms of memory concepts is basically correct, then the data from the standard experiments must represent a mixture of contributions from episodic and categorical memory. On some occasions during a sequence of trials, the subject may recognize a familiar pattern (for example, a run of three consecutive occurrences of the E_1 outcome light in a Humphreys-type experiment) and remember the outcome that previously followed this pattern; on other occasions he may not recognize a familiar pattern but may nonetheless be able to improve his guess concerning the next outcome by using information he has acquired about probabilities of event categories.

In a series of experiments reported elsewhere (Estes, in press-a) I proceeded from this analysis to construct a revision of the usual probability learning experimental paradigm so that one of the component memory processes would be reduced to negligible proportions, thus leaving a clear picture of the other component. The task was presented as a simulation of a public opinion poll preceding an election. The subjects were told that on a series of observation trials they would be presented with simulated data from opinion polls about three pairs of potential candidates (A_1 vs. A_2, A_3 vs. A_4, and A_5 vs. A_6) for a subsequent election. The subjects' task was simply to observe the opinion poll data on the observation trials and attempt to form impressions of the relative likelihood of wins and losses for each of the various candidates. Attending to or rehearsing particular runs of wins or losses would clearly be irrelevant to the task since the subject was tested on his acquired knowledge only on a block of test trials given without feedback following a long series of observation trials. Furthermore, during the test block the subject was also asked to predict election winners from various pairs of candidates (for example, A_1 vs. A_3) that had not been paired during the observation trials. Under these conditions, it was expected that subjects would base their predictions on accrued categorical information concerning the various candidates' relative probabilities of wins and losses.

The results of these experiments seem quite clearly to confirm my conjecture that in the usual probability learning experiment, the subject's performance reflects a mixture of learning categorical frequencies and attempting to learn sequential patterns that actually hinder efficient prediction. With the observation-transfer design, learning proved



TEST DATA

$\pi_i - \pi_j$		% correct
0.04	same	77
	diff.	77
0.08	same	85
	diff.	87

FIGURE 1. Design and results of experiment in which subjects had observation trials on two sets of alternatives, A and B, separately with the π values indicated above, and then tests on new stimulus pairs formed by recombining members of the same set (same) or by pairing a member of Set A with a member of Set B (diff.).

to be more rapid and more precise than has been characteristic in standard probability learning experiments. Even though the differences in event probabilities among the various alternatives were small compared to those usually studied (.62–.38, .58–.42, and .54–.46 for the three observation pairs, respectively), the subjects' proportions of correct predictions on test pairs lined up as a monotone orderly function of the probability differences between the test alternatives, with performance fully as good on new pairs as on observation pairs.

One might wonder whether the subjects were simply learning an ordering of candidates rather than actually acquiring specific information concerning success probabilities. To find out, I investigated an additional variation of the basic design, illustrated in Figure 1. Two subjects were given a series of observation trials on the three pairs of candidates labeled Set A, who had the win–loss probabilities (π values) indicated by the bar diagrams; the subjects were then given observation trials on the three pairs of candidates labeled Set B, these pairs having the same combinations of π values as the corresponding pairs of Set A. Finally, the subjects were given a series of test trials in which test pairs were formed within each set (e.g., A_1 vs. A_2, A_1 vs. A_4) and also across the two sets (e.g., A_1 vs. B_2, A_3 vs. B_5). If the subjects had learned only the ordering within each set they would be helpless on these tests of the A_i versus B_j type.

The test data summarized at the bottom of Figure 1 reveal rather impressive performance. When the members of a test pair differed by .08 in π value, the subjects predicted correctly 85% to 87% of the time, and when the members of a pair differed by only .04, subjects were still correct 77% of the time. Perhaps more strikingly, accuracy of prediction was almost exactly as good for between-sets test pairs (A_i vs. B_j) as for within-set test pairs. Clearly, the subjects had not simply learned orderings of the particular stimuli within each set but rather had acquired information about probabilities of outcomes. It appears that the subjects must have formed representations in memory equivalent to a scale on which the various alternatives (candidates) are placed in positions reflecting their relative probabilities of winning.

Separating Probability from Frequency

With a technique in hand that enables us to trace the course of probability learning, evidently without complication by extraneous activities or processes, we are ready to approach the central problem of what is learned. Progress toward an adequate theory will surely require an adequate characterization of the basis in memory for the predictive behavior that we take as an index of learning. Does the individual in some sense perceive differences or other relations between probabilities, or does he accumulate information concerning either absolute or relative frequencies of events and translate this information into judgments of probability?

The results obtained with the simulated opinion polls bring to mind the concept of "subjective probability," a concept prominent in the literature of decision making (see, e.g., Luce & Suppes, 1965). The subjects did indeed perform on test trials as though they had associated the various candidates with subjective probabilities of winning that accorded quite closely with the objective probabilities. Should we then conclude that the learning going on in this situation can be adequately characterized simply as a translation from objective to subjective probabilities of events?

Ways of testing this characterization are not hard to come by. If it is correct, then learning should be relatively undisturbed if we give the same information relative to outcome probability but vary factors that should be irrelevant to the estimation of probabilities. Our preliminary analysis of probability learning in terms of concepts of memory has suggested that one basis for probability judgments, or predictions, may be the representation of frequencies of event categories in memory. Ordinarily, frequency and probability are totally confounded. In the familiar noncontingent probability learning experiment, for example, an event that has occurred with relative frequency p over a preceding series of trials has approximately probability p of occurring on the next trial. But if we could separate these factors and vary frequency independently of probability, then probability learning would be grossly disturbed if, in fact, the basis for probability judgments is memory of frequency rather than a literal mapping of objective onto subjective probabilities.

The observation-transfer design offers a way to achieve this objective. We can vary the π values, or win–loss probabilities, within

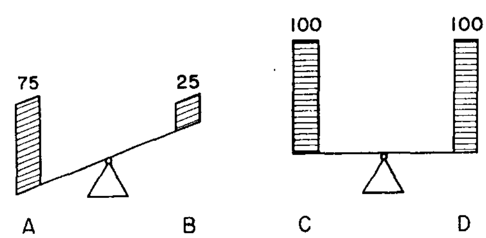


FIGURE 2. Schematic representation of an experiment in which subjects receive 100 observation trials on the stimulus pair AB and 200 trials on CD (with the indicated frequencies of winning outcomes for the four alternatives) and then are tested with A pitted against C.

different observation pairs, independently of the frequency with which the pairs occur during observation trials. Then we can set up transfer tests in which the *frequency* with which a stmulus has been a winner can be pitted in various ways against the *probability* of winning. In Figure 2, for example, imagine that over a series of observation trials the stimulus pair AB has been presented 100 times, and the pair CD, 200 times. Within the pairs, A has a .75 probability of winning over B, but C and D have equal π values of .50. A subject in this hypothetical experiment would have adequate opportunity over the observation series to learn that A has a much higher probability of winning than any of the other alternatives. But if the basis for his judgments of probability lies in past frequencies, there is a complication, in that over the observation series, the stimuli C and D were each winners 100 times and A won only 75 times.

What should we expect the subject to do if we then ask him to predict the result of a test trial that pits A against C? It seems clear that if he bases his prediction on rational grounds he should predict A over C, and at worst, if he is unable to transfer acquired information to the test situation, he should mentally toss a coin and predict A or C with equal probabilities. The outcome seemed sufficiently uncertain to call for a nonhypothetical experiment.

Experiment 1—Effects of Variation in Both Stimulus Frequency and Outcome Probability

Method. The design is shown in Table 1. Three pairs of stimuli were used on the observation trials, with the same combinations of π values used in the experiments cited earlier (Estes, in press-a). But here, in Condition 1 the pairs occurred in the ratio 3:4:2 during observation trials, and in Condition 2, for a different group, they occurred in the ratio 3:2:4. Now, on a transfer test of, say, A versus C following observation trials under Condition 1, we have precisely the situation illustrated in Figure 2: A will have had the higher probability of winning during the observation series but C, the greater total number of winning outcomes.

The stimuli were represented by randomly selected capital letters, displayed on an oscilloscope screen interfaced to a PDP-8/I computer. The situation was described to the subjects as a simulation of a preference survey rather than a pre-election poll. They were told that the computer had been programmed to conduct an imaginary survey to determine the preferences of a sample of people for a number of products of a particular type, for example, headache remedies.

TABLE 1

DESIGN OF EXPERIMENT 1: VARIATION OF STIMULUS FREQUENCY AND OUTCOME PROBABILITY

Independent variable	Pair 1 A	Pair 1 B	Pair 2 C	Pair 2 D	Pair 3 E	Pair 3 F
π value (probability of win)	.62	.38	.58	.42	.54	.46
Relative frequency of pair						
Condition 1	3		4		2	
Condition 2	3		2		4	

On each observation trial, the subject was shown the letters signifying a pair of products. A single tally mark then appeared after the member of the pair preferred by the particular respondent (the "winner" on the given trial). Following a block of observation trials, the subject was presented with various pairs and told to predict which of the two products he would expect to be preferred by a sample of people from the same population that had been surveyed. As in the previous experiments, the subjects received no informational feedback on individual test trials; however, they were told at the end of the test block how many times their predictions had agreed with those generated by the computer on the basis of its knowledge of the true probabilities in the situation.

Groups of 16 subjects were assigned to the two frequency conditions shown in Table 1. Each subject was tested for a single session during which he was exposed to six cycles of observation and test blocks. During an observation block, the three observation pairs occurred in random sequence with the assigned relative frequencies. The constituent stimuli were associated with winning and losing outcomes in accordance with the specified π values. During a test block, each of the 15 stimulus pairs that could be formed from the six members of the observation pairs was presented twice, once in each left-right order, again in random sequence.

Results. The course of learning with respect to the observation pairs themselves was apparently unaffected by the differences in frequency of presentation. The terminal levels of choice of the higher probability alternatives (63%, 64%, and 74%, respectively, for the stimuli with π values .54, .58, and .62) are comparable to those obtained with the same π values but equal stimulus frequencies (Estes, in press-a). Again, these levels were reached after only a few dozen observation trials on each pair.

Of primary interest, however, are the results of those transfer tests for which the

TABLE 2

PERCENTAGES OF CHOICE OF HIGHER PROBABILITY ALTERNATIVES IN HIGH- VS. LOW-FREQUENCY TEST PAIRS OF EXPERIMENT 1

	Frequency combination	
π values	High vs. low	Low vs. high
.58–.46	74	37
.58–.54	73	19
.54–.42	85	36
.46–.42	77	26

TABLE 3

EXPERIMENT 1, CONDITION 1: PERCENTAGES OF CHOICE OF ROW OVER COLUMN STIMULI, ORDERED BY π VALUE

π	.62	.58	.54	.46	.42	.38	Row average
.62		43	63	79	41	69	59
.58	57		73	74	60	71	67
.54	37	27		66	36	45	42
.46	21	26	34		26	33	28
.42	59	40	64	74		63	60
.38	31	29	55	67	37		44

differences in π values were in either the same or the opposite direction from the differences in frequency; these results are presented in Table 2. The four pairs of alternatives involved are listed at the left in terms of their π values. Looking at the choice percentages, the high-versus-low column gives the data for the cases in which the higher probability member of each observation pair also had the higher frequency of occurrence during training. Here probability and frequency are working together, and the result is uniform high preference for the higher probability alternative of each pair. However, in the low-versus-high column are the cases in which the higher probability member of a pair had the lower frequency of occurrence during training—the case illustrated in the introduction to this experiment. The rather striking result is a complete reversal of the subjects' predictive behavior: The percentages of those choosing the higher probability members of the observation pairs are much lower. Thus, when probability was pitted against frequency, in every instance the subjects had a strong tendency to predict that the winner on the transfer test would be the stimulus that occurred more often during the observation phase, even when it had a lower probability of winning than the less frequent stimulus.

The dominating effects of frequency can be brought out in another way by a paired-comparison table showing the pooled results of the transfer tests in terms of percentages of choice of row over column stimuli. In Table 3, with the stimuli ordered in terms of their π values, we find that the orderly progression of values up the columns and across the rows that characterized the earlier experiments is badly perturbed. The reason for the difference is clearly associated with the fact that the stimuli involved in a transfer test may come from pairs that occurred with different frequencies during the observation series. But if we rearrange the same data so that the stimuli are ordered in terms of their total frequencies of winning outcomes over the 144 observation trials given on each pair, then, as shown in Table 4, harmony is restored. Here we find an orderly pattern, indicating that the result of any transfer test can be predicted by knowing the frequencies with which the two stimuli occurred with winning outcomes during training. The results for Condition 2 are entirely similar. On the average, subjects always predict the winner on transfer tests to be the stimulus that accumulated the largest number of wins during the observation series regardless of its *probability* of winning or losing.

These data seem to show that the representations in memory resulting from an individu-

TABLE 4

EXPERIMENT 1, CONDITION 1: PERCENTAGES OF CHOICE OF ROW OVER COLUMN STIMULI, ORDERED BY TOTAL WIN FREQUENCIES OVER OBSERVATION TRIALS

Win frequency	112	90	80	54	52	44	Row average
112		57	60	71	73	74	67
90	43		41	69	63	79	59
80	40	59		63	64	74	60
54	29	31	37		55	67	44
52	27	37	36	45		66	42
44	26	21	26	33	34		28

al's experience with a sequence of events do not at all fit the usual conception of subjective probability. The basis for predictive behavior seems to be not a probability estimate but rather a record in memory of the past frequencies of events—and a record in which information concerning stimulus frequencies trades off with information concerning the likelihoods that various stimuli lead to winning or losing outcomes.

The trade-off certainly cannot be accounted for on the basis of efficiency, for the resulting predictive behavior is uniformly short of optimality and under some circumstances is wildly inaccurate. However, the unexpectedly potent role of stimulus frequency suggests an interpretation of these results in terms common to a great deal of other research. Except for some differences in task orientation, the experiences of subjects in the present experiments are very similar to those of subjects in experiments on verbal discrimination learning and many varieties of recognition learning. Both of these types of learning have been interpreted in terms of the accrual of stimulus frequency information—in effect, the acquisition of stimulus familiarity (Underwood, 1971).

Why then should we not suppose that the critical result of a series of observation trials in a probability learning experiment is a shifting of the positions of the stimuli on a scale of familiarity? We might be slowed down momentarily if someone raised the question of how the subjects managed to learn in the experiments in which all stimuli occurred with equal frequencies. But the same question arose much earlier in connection with verbal discrimination learning, and the answer proposed was that subjects generate differences in item frequency by differential rehearsal. That answer seems fully as plausible in the present context. We need only assume that on observation trials, the subjects have some tendency to rehearse encoded labels for the stimuli and that the principal occasion for this rehearsal is the occurrence of the stimulus together with a winning outcome. Therefore, in the experiments with equal frequencies, stimuli would accumulate rehearsals in proportion to their frequencies of winning outcomes, thus providing the basis for the observed choices on transfer tests. And in the experiments with differential frequencies, variations in frequency of presentation would combine additively with rehearsals to determine the placement of stimuli on the familiarity scale and thus, the results of transfer tests.

This interpretation of the results in terms of differential familiarity also fits in nicely with much time-tested lore about the role of familiarity in such extra-laboratory contexts as politics and marketing. The beginning politician takes part in elections he has no hope of winning, presumably to build up familiarity in the eyes of the voters, which will in turn generate expectation that he may win in future contests. Similarly, advertising campaigns involving lavish use of free samples inveigle consumers into acquiring familiarity with products that have absolutely no objective advantages over their competitors, presumably with the intention of building up familiarity that will lead in the mind of the consumer to increased expectation of beneficial outcome from use of the product.

The interpretation of probability learning in terms of the acquisition of differential familiarity is parsimonious, apparently powerful, and certainly intuitively appealing. But to earn its credentials as a theory, a hypothesis needs not only to integrate existing facts but also to lead to new observations. Thus, the next step required in the series of experiments was to find some way of eliminating the confounding of stimulus and outcome frequencies.

Experiment 2—On the Role of Stimulus Familiarity

Method. One way to eliminate confounding of stimulus and outcome frequencies is illustrated by the modified paradigm in Table 5. To determine whether predictive behavior would reflect variations in stimulus familiarity alone, during observation trials subjects were presented with the type of displays shown in Table 5 for a pair of stimuli A_1 and A_2. The first two displays are for cases already familiar, namely, instances in which A_1 is the winner or A_2 is the winner, these occurring with probabilities π_1 and π_2, respectively. The new feature is the introduction of trials in which the two stimuli are present but with no indication of a winner, the display taking the form shown at the bottom of Table 5,

TABLE 5

TYPES OF OBSERVATION TRIALS IN EXPERIMENT 2

Result	Display	Probability
A_1 wins	A_1 I A_2	π_1
A_2 wins	A_1 A_2 I	π_2
No winner	A_1 ? A_2 ?	$1 - \pi_1 - \pi_2$

with a question mark following each of the alternatives. The subject's task was the same as before except that he was told that on some occasions in the preference survey the results were not available and that in these instances he would see only question marks. However, to make sure that these trials were not ignored, the subject was required to pronounce the names of both alternatives, ostensibly so that the experimenter could be sure the subject had seen them.

The design of the experiment using this modification in procedure is summarized in Table 6. All is much as in the preceding experiment except that now the π values for the members of each observation pair do not necessarily add to unity. The π values do add to unity for A_5 and A_6, but for A_3 and A_4 they only add to .5, meaning that on half of the observation trials when these stimuli are presented, no win or loss is indicated. In the case of A_1 and A_2, the π values add only to .33.

Presentation frequencies and π values were so combined that in a 24-trial observation block, each of the higher probability stimuli A_1, A_3, and A_5 appeared as a winner exactly 3 times, and each of the losing stimuli A_2, A_4, and A_6 appeared as a winner only once. After giving the subjects a series of observation trials under these conditions, a test block was administered to obtain the results of transfer tests in which the frequency of wins was equated, while the stimulus frequency (and therefore presumably familiarity) was varied. Tests were also given in which stimulus frequency was equated but frequency of winning outcomes was varied.

TABLE 6

DESIGN OF EXPERIMENT 2: FREQUENCIES OF WINS AND LOSSES EQUATED OVER PAIRS ON OBSERVATION TRIALS

Independent variable	Pair 1		Pair 2		Pair 3	
	A_1	A_2	A_3	A_4	A_5	A_6
Relative frequency of pair	12		8		4	
π value (probability of winning)	.250	.083	.375	.125	.750	.250
Win frequency	3	1	3	1	3	1

TABLE 7

EXPERIMENT 2: PERCENTAGES OF CHOICE OF ROW OVER COLUMN STIMULI ORDERED BY WIN AND STIMULUS FREQUENCIES

Win frequency/ stimulus frequency	3/12	3/8	3/4	1/12	1/8	1/4
3/12		44	42	72	75	68
3/8	56		50	75	78	75
3/4	58	50		79	81	77
1/12	28	25	21		56	46
1/8	25	22	19	44		45
1/4	32	25	23	54	55	

A group of 24 subjects was given six observation-test cycles under this design; except as noted above, the procedure was the same as in Experiment 1.

Results. Of primary interest are the cases in which only stimulus frequency was varied between members of a test pair. The relevant results are shown in Table 7 for data pooled over all six test blocks. Each entry in the table represents the percentage of choice of the row over the column alternative, when the stimuli are ordered by their win frequencies, and within this category by their stimulus frequencies, on the observation trials.

The results of these comparison seem quite decisive—there is absolutely no sign of any tendency on the part of the subjects to predict that the more familiar of the two stimuli will be the winner when both stimuli have equal previous frequencies of wins and losses. I shall not detail here the results of several other experiments involving variations on this design, but suffice it to say that all of them yield results in full agreement with this one—namely, there is no effect of stimulus familiarity per se when the win–loss frequencies of the stimuli are equated.

The results of the remainder of the transfer tests in this experiment, involving pairs that had different frequencies of winning outcomes on the observation trials, came out much as expected. The pooled data for test pairs pitting stimuli that had three winning outcomes per observation block against stimuli that had one winning outcome per observation block yield an average of 76% correct choices. It should be noted further that within these 3:1 pairs, there was no systematic effect of stimulus frequency.

Finally, if we consider only data from the last two test blocks, the pattern of results reported above is unaltered. The mean percentage of correct choices shown for the pairs with equal win frequencies in Table 7 remains unchanged at 47; and the mean for the 3:1 pairs rises slightly to 81, again with no significant effect of stimulus frequency within these pairs.

With respect to the trade-off between stimulus and outcome frequency, we have evidently disposed of the possible interpretation in terms of stimulus familiarity. The principal alternative direction in which to seek a basis both for the trade-off and for the other properties of predictive behavior revealed by these experiments seems to involve the manner in which events are encoded in memory.

From this latter standpoint, the results suggest a rather curious bias on the part of the subjects with respect to the events they store in memory. Their transfer behavior indicates that they have excellent information concerning relative frequencies of winning outcomes but that they are almost oblivious to the frequencies of losing outcomes for the stimuli they have been observing. This bias, however, might simply be a result of a particular procedure—recall that on observation trials the subjects were always required to pronounce the name of the winner when there was one and to pronounce the names of both alternatives when there was no winner or loser. We must consider the possibility that this procedure leads the subjects to attend selectively to one type of outcome rather than another and thus determines the type of event better represented in memory.

Experiment 3—An Attempt to Vary the Encoding of Trial Outcomes in Memory

Method. In order to check the above interpretation, I designed an experiment with the plan shown in Table 8. The procedure for Group W was identical to that of the preceding study; that is, the subject pronounced the name of the winner when there was one and on other trials pronounced the names of both stimuli. For Group B, the procedure differed in that the subject simply pronounced the names of both stimuli on all observation trials. And, to complete the symmetry, in Group L the subject pronounced the name of the loser when there was one and the names of both stimuli on trials when there was no winner or loser.

TABLE 8

ALTERNATIVES PRONOUNCED ON OBSERVATION TRIALS OF EXPERIMENT 3

Group	Trial type: A B I	Trial type: A B ? ?
B	both	both
W	winner	both
L	loser	both

Three groups of 20 subjects each were given six observation–test cycles with the same procedures as in Experiments 1 and 2 except for the conditions of vocalization of trial outcomes. Half the subjects in each group were given three cycles under Condition 1, in which all observation pairs occurred with equal frequencies, followed by three cycles under Condition 2, in which the stimulus frequencies differed over pairs as indicated in Table 9; the other half of the subjects in each group were given the two conditions in the reverse order. It should be noted that the stimulus frequencies and π values were changed from Condition 1 to Condition 2 in such a way that the frequency of winning outcomes for each stimulus per observation block was unaltered.

Results. For purposes of an overall analysis of the effects of the various independent variables it is necessary to decide on a definition of correct response on each of the test pairs. The most reasonable definition appears to be one based on the relative win frequencies, which are shown for the three observation pairs in Table 9. Recall that in the design of this experiment, the absolute π values changed for two of the observation pairs from Condition 1 to Condition 2 but in

TABLE 9

DESIGN OF 24-TRIAL OBSERVATION BLOCK IN EXPERIMENT 3

Independent variable	Pair 1: A_1	Pair 1: A_2	Pair 2: A_3	Pair 2: A_4	Pair 3: A_5	Pair 3: A_6
Condition 1						
Relative frequency of pair	8		8		8	
π value (probability of winning)	.62	.38	.62	.12	.12	.38
Win frequency	5	3	5	1	1	3
Condition 2						
Relative frequency of pair	12		8		4	
π value (probability of winning)	.42	.25	.62	.12	.25	.75
Win frequency	5	3	5	1	1	3

TABLE 10

VALUES OF r^2 INDICATING PROPORTIONS OF VARIANCE IN TERMINAL CHOICE DATA (BLOCKS 4–6) OF EXPERIMENT 3 PREDICTABLE FROM THREE INDEPENDENT VARIABLES

Group	Wins	Losses	Relative π values
B	.74	.13	.61
W	.83	.09	.61
L	.13	.32	.25

such a way that the frequencies of wins and losses per observation block did not change; consequently, the relative π values, defined as the probability of a win on any observation trial that eventuated in either a win or a loss, did not change either. The subjects had no basis for any supposition other than that the stimulus alternatives had been assigned at random to π values; on this assumption the rational course of action would be to make predictions on the various tests in accord with the relative π values. Using this definition, one can order the alternatives A_3, A_6, A_1, A_2, A_5, and A_4; then, for any test pair, the correct prediction will be that the higher ranking member of the pair will be the winner.

An analysis of variance conducted on the choice data scored in terms of correct response frequencies showed, first of all, that the subjects were almost oblivious to shifts in absolute π values; the main effect of conditions yielded an F less than unity, and the same was true for the interaction of Conditions × Groups. Consequently, the data were pooled over Conditions 1 and 2 for all subsequent analyses.

A bit more surprisingly, perhaps, the main effect of groups also proved nonsignificant, $F(2, 54) = 2.44$. However, the interaction of groups with specific test pairs was significant, $F(28, 756) = 5.37$, $p < .01$. When the relative frequency of losing outcomes was small for the correct alternative of a test pair, there were no consistent differences among the three vocalization groups. But as this frequency increased, the functions relating proportions of correct choices to relative frequency of winning outcomes on the correct alternative diverged and tended to line up in order, with Group L much the lowest and Group W the highest.

The differences among the vocalization groups are brought out most clearly by model-oriented analyses, to be presented in a later section. However, correlational statistics cautiously interpreted can also be instructive, as may be seen in Table 10. It is clear first of all that for each group, information concerning the frequency either of wins or of losses during observation trials provides a better predictor of choice behavior than the relative π values. But the frequency of wins accounts for a substantial part of the variance in the choice data only if the subjects pronounce the names of winners on observation trials, and the frequency of losses is a major determinant only when the subjects pronounce the names of losers but not winners. However, these relationships cannot be attributed to effects of pronunciation per se. As a consequence of the design of this experiment with respect to probabilities of wins, losses, and "blank trials," the frequencies of pronunciation of names of various stimuli in an observation block are imperfectly correlated with the frequencies of wins and losses. When correlations are computed between choice percentages and relative frequencies of pronunciation of stimulus names over the various test pairs, the values of r^2 obtained range from an average of .01 for Group B to .06 for Group W and .24 for Group L.

Since frequency of vocalization per se is virtually unrelated to choice performance, it seems a reasonable hypothesis that instructions to pronounce a given type of outcome lead subjects either to attend selectively or to rehearse preferentially the names of stimuli leading to that type of outcome. In either case, pronouncing the names of winners would increase the likelihood that encoded representations of stimuli receiving winning outcomes would be stored in memory, and similarly for losers.

The asymmetry in the correlations for Group B is not unexpected on the basis of evidence from other studies (Bush & Mosteller, 1955; Estes, 1959) and suggests that unless constrained by special instructions to attend to losing outcomes, subjects tend to ignore losses, store information in memory almost exclusively in terms of relative frequency of winning outcomes, and make predictions on the basis of this stored information.

On the Storage of Absolute Versus Relative Frequency Information

Given the conclusion that the individual encodes and stores information regarding stimulus-outcome categories to which his attention has been directed by the task orientation and procedures, the principal remaining question bearing on the class of admissible models is that of how categorical information accrues over observation trials. In particular, we need some indication whether absolute or only relative frequency information is stored in memory.

The first possibility, storage of absolute frequency information, is suggested by the counter model of Ekstrand, Wallace, and Underwood (1966), or by Restle's schema theory (1961). In either of these models, it is assumed that distinct representations of each occurrence of an event combination belonging to a given category are stored in memory. Thus, when an individual has to make a probability estimate or a prediction, he would presumably be able to form a ratio of the absolute frequencies of the event representations for the relevant categories and convert this ratio to a probability estimate that would then be the basis for his response.

The alternative possibility, that only information on relative frequencies, or ratios, is stored, is suggested by stimulus-sampling theory. In effect, an indicator is set up in the memory system for each category to which the individual attends, with the value of the indicator falling in the interval from 0 to 1 and being adjusted with each occurrence of an event of the given category.

The notion of a relative frequency indicator can be given a direct interpretation in terms of stimulus-sampling theory. We need only assume that in any situation there is a fixed population of potentially available background contextual cues, a random sample of which are in an active state on any given trial and become associated with the category to which the individual attends. The proportion of these contextual cues associated with a given category would then constitute the indicator, and its value would change from trial to trial in accord with the linear operators of stimulus-sampling models.

The course of acquisition of information over a series of observation trials, as evidenced in predictive behavior on transfer tests, might be quite similar for models based on either absolute frequency or relative frequency assumptions. However, it should differ for the two models at the point of a shift in event frequencies. If absolute frequency information is being accumulated, with increasing numbers of preshift trials the frequencies attaching to the various event categories will become large—consequently, a large number of postshift trials will be required to yield a detectible change in performance. (The same implication would follow from a cumulative strength model such as the "beta model" of Luce [1959].) In contrast, if relative frequency information is being acquired (at least as interpreted in terms of stimulus-sampling theory), the relative frequency indicator will be adjusted on each observation trial by a constant fraction of the difference between its current value and either unity or zero. Consequently, the rate of acquisition of new information following a shift in frequencies will be independent of the number of preshift trials. The following experiment was thus contrived to provide some information on which to base a choice between the absolute and relative conceptions.

Experiment 4—Effect of Early Versus Late Shifts in Stimulus Frequencies

Method. The procedures were similar to those of Experiment 1 except that the relative frequencies of occurrence of the different observation pairs were shifted part way through the experiment. One group of 20 subjects was assigned to an early-shift condition, and another group of 20 subjects, to a late-shift condition. Both groups were given six cycles of observation and test trials in which the π values for the three training stimulus pairs were held constant at .62–.38, .58–.42, and .54–.46, respectively. In Phase 1, all three training pairs occurred equally often in each observation cycle; consequently, the stimulus alternatives fell in the same positions when ordered either by frequencies of winning outcomes or by π values, as indicated in Table 11. In Phase

TABLE 11

DESIGN OF EXPERIMENT 4: EFFECT OF EARLY VERSUS LATE SHIFTS IN STIMULUS FREQUENCY

Independent variable	Pair 1		Pair 2		Pair 3	
	A_1	A_2	A_3	A_4	A_5	A_6
π value (probability of winning)	.62	.38	.58	.42	.54	.46
Relative frequency of pair						
Phase 1	24		24		24	
Phase 2	24		12		48	
Win frequency						
Phase 1	15	9	14	10	13	11
Phase 2	15	9	7	5	26	22

2, the relative frequencies of occurrence of the training pairs A_3A_4 and A_5A_6 were shifted to new values; thus, in Phase 2, the order of the stimulus alternatives in terms of win frequencies was no longer the same as the order in terms of π values. For the early-shift group, the shift in frequencies (Phase 2) occurred following the first observation–test cycle; for the late-shift group it occurred following the third cycle.

Results. Consider now how our predictions concerning test performance on a particular pair, say A_3A_5, during Phase 2 would differ for the absolute and relative models. During Phase 1 these two alternatives had nearly equal frequencies of winning outcomes (A_3 having a slight advantage), but during Phase 2, A_5 had nearly four times as many winning outcomes as A_3. Thus, one would expect choices on test trials during Phase 1 to yield approximately .50–.50 preferences, whereas during Phase 2 one would expect subjects to select A_5 most of the time from this test pair. Now, according to an absolute model, the subject's record in memory of his experience with the various alternatives at the end of Phase 1 will have the form of "counts," that is, records of the approximate total frequencies of winning outcomes. These counts will be approximately equal for A_3 and A_5, but for the early-shift group they will both be small numbers, whereas for the late-shift group they will be much larger numbers. Consequently, for the early-shift group only a few trials under the new conditions of Phase 2 would be required to change the ratio of the counts on the two alternatives substantially and yield a noticeable change in test performance. But for the late-shift subjects, who start with larger counts on the two alternatives, a much larger number of trials in Phase 2 would be required to produce a change in the ratio of these counts large enough to yield a noticeable change in performance.

In general, then, the prediction from the absolute model must be that the rate of learning during Phase 2 will be slower, the larger the number of trials given during Phase 1. Although I shall not go through the detailed reasoning here (see, e.g., Estes & Straughan, 1954), the prediction from the relative model must be that the rate of learning during Phase 2 will be independent of the number of trials given in Phase 1.

The results of primary interest are the learning curves over the postshift trials; these are shown in Figure 3 for the early- and late-shift groups with the data pooled over all 15 test pairs in each case. The figure represents performance on test cycles 2–4 for the early-shift group and on cycles 4–6 for the late-shift group. Performance for the two groups is very similar on the last preshift block (as expected, since conditions had been the same up to this point) and continues to be strikingly similar over the postshift trials regardless of the point of the shift. An analysis of variance yielded an F less than unity for the mean difference between early- and late-shift groups over the postshift trials. In spite of the difficulties always associated with accepting a null hypothesis, we must at the least conclude that there is no positive sup-

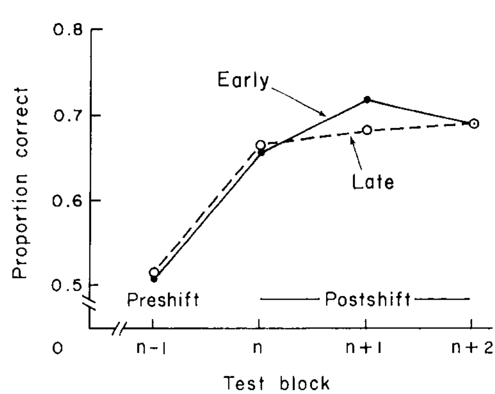


FIGURE 3. Correct choice proportions on transfer tests following early versus late shifts in relative frequencies of events.

port in these data for a model of the absolute type.

General Discussion: Interpretations and Applications

Summary of Major Findings

Our preliminary examination of the present state of knowledge concerning probability learning pointed up the need for untangling the varying contributions of learning and performance variables to the patterns of results obtained with different subject populations and different experimental procedures. Accordingly, throughout the present series of studies I have tried to reduce variance attributable to various determinants of performance by simplifying the subject's decision problem, and thus to bring out in clearer relief the information-processing components of probability learning.

The periods within each experiment during which the subjects were able to obtain information concerning event probabilities were segregated from the periods during which they were tested for their ability to predict events, thus minimizing subjects' tendency to treat the experiment as a guessing game. Furthermore, subjects were always fully instructed concerning the probabilistic nature of the task, and they were encouraged to base their choices on the information about event probability that they had accumulated from their observations of sequences of trial outcomes.

Under these circumstances, learning proved to be extremely rapid and precise compared to that usually found in classical probability learning experiments, even though the present experiments generally presented more difficult tasks in terms of the numbers of probabilistic combinations the subjects had to deal with simultaneously.

But at the same time, the results suggest that the term "probability learning" is in a sense a misnomer. I have found nothing to encourage the tendency to think of probability learning as a basic or unitary process or as a direct manifestation of a capacity for perceiving the statistical structure of sequences of events. The subjects clearly are extremely efficient at acquiring information concerning relative frequencies of events. But by appropriately modifying the instructions on how subjects are to respond on individual observation trials, I have shown that they acquire this information selectively about the events to which they attend. The result is that under some circumstances, the information subjects acquire leads them to make judgments that appear to reflect differences in probabilities of events with great fidelity, but under slightly different circumstances, equally efficient operation of the same learning process leads them to make judgments of likelihoods of events that are widely at variance with the actual probabilities.

The apparent trade-off between stimulus frequency and outcome probability observed in a number of the present experiments might be taken to indicate that subjects confuse familiarity of a stimulus with the probability that it leads to a winning outcome. However, appropriate controls showed that stimulus frequency per se does not influence choice probability, the apparent effects being attributable to inefficiency or bias in the process of encoding events in memory.

The best characterization of the learning process I can offer at present is that subjects categorize the events involved in a task and then learn relative frequencies within classes. However, whether owing to lack of adequate training or to limitations of memory capacity, they do not always carry the process of categorization far enough for optimal performance on a given task.

Consider, for example, the situation in which the task involves a simulated public opinion poll with several pairs of candidates for an election running against each other. In acquiring information concerning the results of the simulated preference survey, the subjects clearly categorize the trial outcomes appropriately in terms of wins and losses for individual alternatives, but they generally fail to assign to separate categories the data belonging to the pairs of stimuli that have been pitted against each other during the observation series. If the different pairs are represented equally often during the observation series, which has been almost invariably the case in previous studies of probability learning, this lack of second-order categorization leads to no error. But if we modify the usual procedure and present different pairs with different frequencies, subjects make very large errors of judgment, in some instances predicting that an alternative that has appeared frequently but as a consistent loser will be preferred to another alternative that has appeared infrequently but has been a consistent winner.

Information and Performance

Given our conclusion that probability learning is a derivative of frequency learning, how do we conceive the connection the learner makes between his state of frequency information and his predictive responses, or judgments of probability? I suggest, first of all, that there is no immutable general rule but rather that individuals bring frequency information to bear upon specific problems in accord with task requirements and relevant experience.

In situations of the type investigated in the present series of experiments, it appears that the learner translates the request for predictions based on probabilities into a request for relative frequency judgments. Thus, when an individual has had experience with a series of opinion polls and then is asked to predict the result of a preference test or an election, he predicts success for the alternative that he remembers as having had the greatest frequency of success in his past experience.

If, as in many classical probability learning experiments, the individual is led to believe that he is dealing with a problem situation that has a determinate solution, then I see no reason to assume that his trial-to-trial predictions will reflect trial-to-trial assessments of relative frequency. Rather, in the light of existing evidence on all-or-none learning (Estes, 1964; Restle, 1965), we might better assume that in situations of this type, the learner seeks ways of encoding recurring patterns of events in context, ways that enable him to base predictions on episodic memory of specific circumstances under which a to-be-predicted event occurred in the past.

However, this response system does not remain static or immune to effects of changing experience. During the series of trials in which an individual is responding in accord with these encodings, we might say at a descriptive level that he is using a response *strategy* as this term has been used, for example, by Restle (1962). Nonetheless, learning in the sense of accrual of relative frequency information continues. At some point, most likely following an error or a sequence of errors, the learner might be expected to reassess his state of frequency information and, if necessary, recode the events and thus change his response strategy.

Because the relationships between states of information and response rules are not well understood, it seems quite possible that various comparisons made in the probability learning literature, especially those involving different developmental levels, may be misleading. Consider, for example, the finding that in probability learning experiments younger children tend to show a strategy of maximizing successes, presumably the most rational approach to the situation, whereas older children tend to yield probability matching (Weir, 1964). It is possible, though, that the younger children are actually operating at a more primitive level, tending to ignore negative outcomes and to make choices directly reflecting their impressions of the frequency of positive outcomes, and that the older children are actually operating at a more sophisticated level, attending to both positive and negative outcomes and encoding events in terms of their information concerning relative frequencies of both. If this analysis is correct, then it follows that meaningful developmental comparisons will require experimental procedures that will lead children of different ages to follow similar response strategies and that will thus yield interpretable data reflecting differences in the nature or efficiency of learning in the sense of information processing.

Some Implications for Probability Learning Outside the Laboratory

In terms of the revised theoretical formulation taking shape as a result of these new investigations, it appears that we can begin to make sense of some of the otherwise puzzling observations on characteristic human behavior in probabilistic situations outside the laboratory (Jenkins & Ward, 1965; Kahneman & Tversky, 1972; Smedslund, 1963; Brehmer, Note 1).

If we accept the idea that probability learning is actually based on the acquisition of information about frequency of various individual events occurring in a probabilistic situation, we can state two general conditions that must be met in any situation if the learning process is to lead to veridical estimates of probability by the learner: (a) The alternative events involved in a situation must have equal opportunities of occurrence and (b) the learner must attend to and encode occurrences of all of the alternative events with equal uniformity or efficiency.

Both of these conditions are probably satisfied quite well, for example, in learning to anticipate changes in the weather. Shifts toward both fair and foul weather have equal opportunities of occurrence, and quite likely both eventualities are clearly perceived and encoded by the human observer. However, in many other situations, these conditions must be uniformly and grossly violated. For example, in the primaries preceding a general election, it is common for different candidates to take part in different numbers of primaries. Similarly, in an individual's experience with alternative treatments or remedies for illnesses, it is probably most common for his experience with different subsets of remedies to be quite unequal, owing to the effects of advertising, hearsay, or simply habits of trying particular remedies. In these situations we must expect that learners will be almost entirely unable to correct for differences in numbers of opportunities and thus will often be misled in the probability estimates they form on the basis of experience. On the whole, they will tend to persist in choosing the more familiar candidate or remedy, not because of its familiarity per se, but because its more frequent occurrence in their experience has given it an opportunity to accumulate a greater total number of successes than its competitors that have been experienced less often.

Just as clearly, the requirement of equivalent attention to alternative outcomes is systematically violated in many practical situations. Conspicuous examples of this occur when individuals form impressions regarding the probability of crime in different localities or the probability of accidents using different modes of transportation. The occurrence of a crime or an accident is a clearly perceptible and readily encodable event that will inevitably leave its residue in the memory system. But a basic problem in each case, from the standpoint of the present analysis, is that there is no correspondingly clear-cut way for the learner to identify the individual occasions when there were opportunities for crimes or accidents to occur but they did not in fact transpire, that is, when there were negative outcomes. Thus, we might expect automobile drivers in a given locality to form very accurate impressions of the relative frequencies of accidents on the thruway and on a nearby city street but at the same time to have grossly distorted conceptions of the probability of having an accident during a given number of miles traveled in the two cases.

In general, we must evidently say that the term *probability learning* characterizes a type of problem situation rather than a type of learning. One and the same set of underlying processes can be expected to lead to highly efficient and veridical probability learning under some circumstances but to systematic and often gross distortions of probability estimates in others. Nothing in our analysis leads us to expect that it should be easy to train people to judge probabilities accurately in a wide range of practical situations, but it may still be possible to make progress in this direction once the nature of the problem and the processes of learning and performance are conceptualized within a satisfactory theoretical framework.

Toward a Theory of Probability Learning Based on Concepts of Memory

Characteristics of the Acquisition Process

In the earlier work on probability learning, the predominant strategy was to apply various highly specific mathematical or, in some cases, computer-simulation models to data, in the hope that one of them might prove to account for the detailed course of acquisition over varying experimental conditions. As I have noted above, it is apparent that many of these specific models do very well under particular circumstances, but none comes close to providing the desired generality. Consequently, in this essay I shall explore the alternative strategy of drawing upon a wide range of data for clues to the *type* of model that might have some generality. In this section I propose only to summarize where we stand in this respect and to present an example of a model that both accounts for the new information we have acquired and may help to bring out some theoretical connections between quite different types of experiments, though at the cost of not providing full and detailed quantitative accounts of the data of any one experiment.

First, let us summarize some of the salient characteristics of the probability learning process that must evidently characterize an adequate model.

1. When the learner is tested in the same contexts in which he has had an opportunity to make observations, and in particular when the situation involves only a few frequently recurring contexts, the learning of associations between outcomes and contextual patterns appears to be all-or-none in character.

2. When tests are given under contextual conditions quite different from those of observation trials (as in the experiments described in this article), learning appears to be analog in character, with even very small differences in event probabilities being reflected in subjects' predictive tendencies.

3. Transfer performance following a sequence of multiple-cue probability learning trials has many of the properties that would be expected if the learner had formed a representation in memory of a scale on which different choice alternatives were positioned according to their frequencies of occurrence. These scale values appear not to represent absolute frequencies, but rather the frequencies with which instances of stimulus-outcome categories occur relative to instances of all other categories that occur in the given situation.

4. The event categories whose frequencies come to be represented in memory are those to which the individual is led to attend by the task orientation and training procedures.

5. Changes in scale values following a shift in event probabilities appear to be described by the learning operators characteristic of stimulus-sampling models rather than those of accumulative models such as Luce's beta model (1959) or the frequency counter model of Ekstrand, Wallace, and Underwood (1966).

Modes of Information Storage and Retrieval

As a first step toward formalization, I propose to consider the possibility of meeting the requirements sketched above with an associative theory of memory that takes account of the newer concepts of organization and retrieval strategies. I shall organize this discussion in terms of a particular variant, the associative coding model (Estes, 1973, in press-b). This model shares a number of ideas with Feigenbaum (1963) and Anderson and Bower (1973) with respect to the associative structure, with Johnson (1970) in regard to coding, and with Tulving (1968) concerning the conception of retrieval cues.

The basic information storage process in this model is conceived to be the formation of a memory trace representing the occurrence of an event in context. If an event E occurs in a context x, it is not assumed that an association forms between E and x directly, but rather that they become associated by way of a common control element C, as illustrated in Figure 4. This associative structure has the property that future reinstatement of the combination xE will activate the control element and hence any other representation in memory with which this control element is in turn associated. This associative unit may be more compactly represented by the notation T_{xE}. In a two-choice probability learning situation, two types of events occur—E_1 and E_2 (which in the simulated preference surveys would be wins and losses, respectively)—and thus two types of traces would be stored, T_{xE_1} and T_{xE_2}. On a test trial, the test context would reactivate a trace of one type or the other and lead to the corresponding predictive response on the part of the subject.

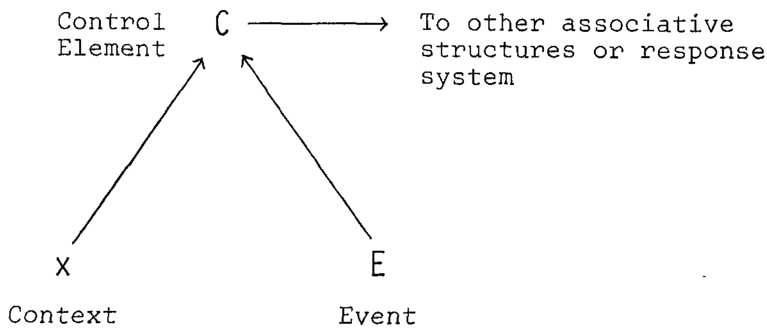


FIGURE 4. Schematic representation of associative memory structure representing the occurrence of an event E in a context x.

Three types of contextual cues need to be distinguished, together with the possibility that memory storage may occur with respect to any of these, or even to all three types concurrently. These are (a) background cues that remain functional regardless of variations in event schedules, (b) local context, specific to a particular event schedule (including, for example, stimulus patterns associated with runs of like outcomes over successive trials), and (c) stimulation arising from internally generated "coding responses." The second and third types are assumed by virtually all theorists (see, e.g., Myers, in press) to be of major importance in standard, two-choice prediction experiments; under noncontingent probability schedules, these are typically characterized by a relatively small number of frequently recurring event patterns (e.g., short runs, double alternations) to which subjects are known to attend (Feldman & Hanna, 1966). In the transfer design of the experiments reported in this article, local contextual cues arising from the training event sequences would not be present during transfer tests; therefore, test performance must be presumed to depend on memory traces associating outcome events with background cues.

Furthermore, there is evidence that under some circumstances subjects may shift attention from one type of context to another. Thus, Mandler, Cowan, and Gold (1964) observed that, in a concept learning experiment including correlated, partially valid cues, subjects exhibited probability matching with respect to the partially valid cues during the presolution period, but nonetheless went on to achieve 100% correct responding to the fully valid cues.

It is quite possible, and even likely, that subjects may simultaneously acquire information relating outcome events to both background and local context. This multiple processing may not be apparent during learning under a particular event schedule if subjects typically select one of the available types of context as a basis for responding, but it may emerge with a change in conditions. A case in point is observed when, following a series of trials on a standard two-choice, noncontingent probability schedule, subjects are suddenly shifted to extinction ("blank trials," as in Neimark, 1953) or to a noncontingent success schedule (all responses correct, as in Yellott, 1969). Typically, subjects continue to respond at a probability matching level, even though there is no continuing feedback to support this predictive behavior.

We know that the data for predictive behavior during preshift trials are well described by a pattern model that assumes all-or-none learning of coded patterns of contextual cues that are available during the noncontingent series, but these sequential patterns must suddenly become unavailable after the shift to no-feedback or noncontingent success conditions. Since, nonetheless, probability matching performance continues, evidently we must assume that learning also occurred during the noncontingent series with respect to background cues that remain available following the shift.

Even more direct evidence for memory storage on multiple tracks is available from studies reported by Binder and his associates (Binder & Estes, 1966; Binder & Feldman, 1960) conducted with a transfer paradigm. In these studies, training was given with a modified paired-associate procedure in which patterns of cues were associated with outcome events according to the schema AB–E_1, AC–E_2, DB–E_3, DC–E_4, but some of the patterns occurred more often than others so that, for example, AB–E_1 might occur twice as frequently as AC–E_2 during training. During the learning sequence, conducted with an anticipation procedure, the subjects quickly arrived at an asymptotic level of 100% correct performance, indicating that they had stored memory traces relating the AB pattern to event E_1, the AC pattern to event E_2, etc. and were responding on the basis of retrieval of the corresponding traces.

But on subsequent transfer tests, when for example, Cue A was presented alone, it was observed that the subjects predicted event E_1 with a probability that matched the relative frequency with which AB and AC occurred during training. Thus, although it was not manifest during the training series, the subjects must also have been storing traces relating the individual cues such as A or B in their background context to the outcome event.

The picture to which we appear to be led is one of multiple-track learning in which memory traces may be formed concurrently at a number of levels of processing or coding of contextual information. Differing test conditions lead to the retrieval of different types of contextual trace patterns and consequently, different patterns of test performance.

Episodic Memory and the Pattern Model

On the basis of a variety of considerations discussed in preceding sections, it seems reasonable to assume that predictive behavior is dominated by episodic memory of patterns of events whenever this mechanism is available. Two cases need to be distinguished, corresponding to short-term and long-term retention. The more obvious but, in practice, less important case is that in which a situation is reinstated following a very short interval, so that the individual has a relatively full representation of the situation and the outcome in short-term memory and simply predicts that the same outcome will occur again. The more important case arises following a longer retention interval, when the individual cannot remember all of the original circumstances but may be able to recall a code or label that he applied to the original episode—therefore, he predicts the outcome that he associated in memory with the given code or label.

With some simplifying assumptions, this latter conception leads to a simple, but still surprisingly powerful, model for predictive behavior in the standard, noncontingent probability learning situation. The special condition for applicability of the pattern model is that the sequence of learning trials include a number of recurring spatial or temporal patterns of stimuli for which the individual has already established coding responses, usually verbal labels. On a trial when a code c is activated and an outcome E_i occurs, a memory trace T_{cE_i} is formed. When the same code is reinstated by the context of a later trial, the trace is reactivated, and any response (in this context a predictive response) associated with the event E_i is evoked.

A complication arises if the same code c is available both on a trial when event E_i is the outcome and on a subsequent trial when a different event E_j is the outcome—it thus enters into two trace structures, T_{cE_i} and T_{cE_j}, that tend to evoke different responses. In previous developments of the associative coding model (Estes, 1973), I assumed that if the responses to E_i and E_j were mutually exclusive, then the one more recently activated would inhibit the other through an inhibitory association.

On the simplifying assumptions that a fixed set of N coding responses is available throughout an experiment and that the different codes are equally likely to be available on any trial, this special case of the associative coding model is formally equivalent to the pattern model as applied to a number of variants of simple probability learning by Estes (1959), Suppes and Atkinson (1960), and Yellott (1969).

Categorical Memory for Relative Frequencies

Whether coding responses are available or not in a given situation, memory traces must be formed associating background context with stimulus events to which the individual attends. The sample of background cues receiving attention will vary from trial to trial; thus, for example, in a sequence of observation trials in a probability learning experiment, traces T_{xE_1}, $T_{x'E_1}$, $T_{x''E_2}$, . . . might be established ($x^{(i)}$ denoting the sample of background context and E_1 and E_2 denoting the outcomes, e.g., A_1-Win or A_2-Loss).

On a test trial, the subject will be exposed to a sample of contextual cues that, in general, will have elements in common with a number of the different samples that were present on the observation trials. Consequently, a number of the traces may be partially reactivated, but a match between the test context and any of the traces will be impossible since the outcome events are missing from the test context. How, then, can a predictive response (or a probability or frequency judgment) be generated? The answer, I propose is to be found in the scanning model of stimulus-sampling theory (Bower, 1959; Estes, 1962, 1966).

The scanning model for the selection of predictive responses. Interpreted in terms of the present problem, the scanning model implies that in a test situation, the individual interrogates his ensemble of partially activated memory traces by generating probes that take the form of coded representations (labels or the equivalent) of the alternative to-be-predicted events. If the stimulus input from one of these probes, together with the current sample of contextual cues presented by the test situation, matches one of the traces $T_{x^{(i)}E_j}$, then the individual predicts outcome E_j.

If the scanning is done in a random order, then the probability that a match will first occur on a trace including E_j is simply equal to the proportion of traces in the ensemble scanned that have the component E_j. Consider first a special case in which the individual has attended only to winning outcomes. On a test of A_i versus A_j, his probability of predicting a win for A_i would be simply

$$P_{ij}(i) = \frac{W_i}{W_i + W_j} , \qquad (1)$$

where W_i and W_j denote the frequencies of winning outcomes for the two alternatives during the observation series. Similarly, if he attended only to losing outcomes, the probability of predicting a loss for A_j, and therefore a win for A_i, would be

$$P_{ij}(i) = \frac{L_j}{L_i + L_j} , \qquad (2)$$

where L_i and L_j denote frequencies of losses.

Under some circumstances, it would be reasonable to apply Equation 1 or 2 directly. (See, for example, the treatment of relative frequency judgments in a later section.) However, there is substantial reason to think that in normal adult choice behavior, alternatives are not scanned at random; rather, the individual tends to first scan the alternative he has most recently chosen. With this assumption incorporated, the expressions for choice of alternative A_i over A_j take the forms (cf. Estes, 1960, 1962)

$$P_{ij}(i) = \frac{W_i^2}{W_i^2 + W_j^2} \qquad (3)$$

and

$$P_{ij}(i) = \frac{L_j^2}{L_i^2 + L_j^2} . \qquad (4)$$

An additional possibility to be considered is that the individual attends to both winning and losing outcomes. In this event, the simplest assumption would be that on a test of A_i versus A_j, the individual scans both alternatives, stopping the scan if he recalls a win for A_i and loss for A_j or a loss for A_i and win for A_j, but continuing if he recalls wins for both or losses for both. The probability of a choice of A_i over A_j would then be

$$P_{ij}(i) = \frac{W_iL_j}{W_iL_j + W_jL_i} . \qquad (5)$$

Before trying to apply these functions to data, we should note that Equations 1 through 5 all assume an asymptotic state of learning in which every possible sample of test context will find a match in the ensemble of stored memory traces. To allow for incomplete learning, it will suffice for our present purposes to introduce a parameter ϕ to denote the proportion of instances in which a match between a test context and a memory trace will be available. The value of ϕ will vary from 0, prior to the first observation trial, to 1, at the asymptote of learning. Thus, on any test trial there will be probability $1 - \phi$ that no memory match will be found and that the individual will have to choose on a chance basis. The predictive equations will take the forms

$$P_{ij}(i) = (1 - \phi)(.5) + \phi\frac{W_i^2}{W_i^2 + W_j^2}, \quad (6)$$

$$P_{ij}(i) = (1 - \phi)(.5) + \phi\frac{L_j^2}{L_i^2 + L_j^2}, \quad (7)$$

and

$$P_{ij}(i) = (1 - \phi)(.5) + \phi\frac{W_i L_j}{W_i L_j + W_j L_i}, \quad (8)$$

for the cases when the individual scans on the basis of memory traces for wins, losses, or both, respectively.

Elsewhere (Estes, 1960, 1962), I have discussed more elaborate machinery for dealing with trial-to-trial changes in choice probability during learning. For present purposes, in the case of any experimental application, we need only determine the value of ϕ from the observed choice data on any one pair of alternatives at a given stage of learning; this determination presents no difficulties since all of the other quantities entering into Equations 6 through 8 are observable.

In order to make predictions about a given experiment, we must hypothesize which type of outcome the subjects are attending to and encoding. Alternatively, we can put the model comprising Equations 6 through 8 into a form enabling us to interrogate the data and obtain evidence as to what the subjects are doing. For this latter purpose, we need only define parameters w, x, and b, denoting the proportions of subjects who encode and scan wins, or losses, or both, respectively, under a given condition and combine Equations 6 through 8 into the single function

$$P_{ij}(i) = (1 - \phi)(.5) + \phi\left[w\frac{W_i^2}{W_i^2 + W_j^2} + x\frac{L_j^2}{L_i^2 + L_j^2} + b\frac{W_i L_j}{W_i L_j + W_j L_i}\right]. \quad (9)$$

Since, in the observation-transfer experiments with which we are concerned, data are available for a relatively large number of test pairs, we can, in effect, use a portion of the data to evaluate the parameters ϕ, w, x, and b (actually there are only three free parameters, since we must have $w + x + b = 1$) and then predict the data for the remainder of the test pairs. A more systematic procedure, and the one I shall follow, is to evaluate the parameters by a least squares fit of Equation 9 to the entire set of test pairs for a given experiment simultaneously.

Application of the model to multiple-cue probability learning. In the case of Experiment 1, since we have already seen substantial evidence that the subjects were responding primarily on the basis of the relative frequencies of winning outcomes on the various alternatives, we should expect our estimation procedure to yield a value near unity for the parameter w in Equation 9. This expectation is confirmed: Least squares determinations on the data of Tables 3 and 4 (and the corresponding data for Condition 2) yield estimates of $\phi = .95$, $w = .97$, and $x = 0$ for Condition 1, and of $\phi = .99$, $w = .94$, and $x = .01$ for Condition 2. The very high estimates of ϕ indicate that learning was very rapid, with nearly all of the data coming from the asymptotic state. A concise picture of the goodness of fit of the model can be obtained by averaging the values in each row of the paired-comparison tables—for each alternative this yields the mean proportion of times it was preferred over all other alternatives in the set. The predicted and observed paired-comparison scale values so obtained are shown in Figure 5.

The data of Experiment 2 do not provide much of an exercise for the model because of the large number of duplicate values of win and loss frequencies. In the paired-comparison values of Table 7, no systematic trends are predicted with respect to stimulus frequencies. Values in the upper left and lower right quadrants (representing relative win frequencies of 3 vs. 3 and 1 vs. 1, respectively) should vary around a mean of 50%, which they do rather closely. Values in the upper right (3 vs. 1) and lower left (1 vs. 3) quadrants should vary around means between 50%–100% in the former case and between 0%–50% in the latter case. With a ϕ estimate of .61, the predicted means are 74% and 26%, respectively.

The most instructive application of the model is to Experiment 3, in which differential instructions for pronouncing names of outcomes on observation trials were expected to modify subjects' tendencies to encode winning and losing outcomes. Here again, I used data for the full, paired-comparison matrices to estimate the parameters of the model. In all cases, the value of ϕ was relatively high (.80, .90, and .71 for Groups B, W, and L, respectively), indicating near asymptotic performance. As anticipated from our knowledge of the instructions, estimated values of w in Equation 9 were much higher for Groups B and W (.71 and .90, respectively) than for Group L (.38). Conversely, values obtained

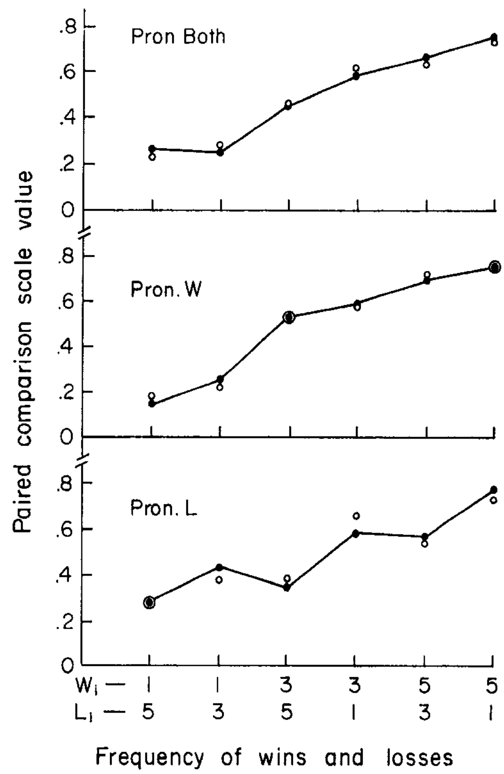


FIGURE 6. Observed and predicted paired-comparison scale values for the three pronunciation groups of Experiment 3.

for x, reflecting the relative attention to losing outcomes, were relatively low for Groups B and W (.29 and .10) and much higher for Group L (.62).

Mean observed and predicted paired-comparison scale values were computed for each group (as done previously for the data of Experiment 1); these are plotted in Figure 6. The figure not only shows the relatively good fit of model to data, but brings out again the effects of the pronunciation conditions on information processing. The points on the abcissa of Figure 6 are ordered first by increasing relative frequency of winning outcomes per alternative on observation trials (W_i), and then, within each value of W_i, by decreasing relative frequency of losing outcomes. Between the pairs of points on the abcissa associated with decreases in loss frequency (e.g., from 5 to 3 between the first and second points), one can see that the scale values for Group W uniformly increase,

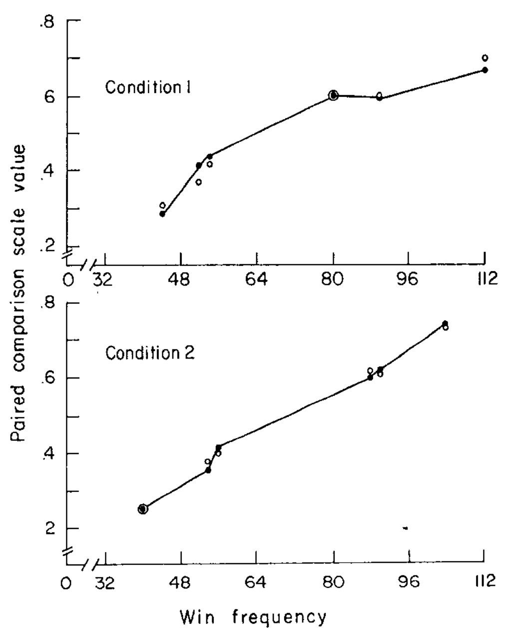


FIGURE 5. Observed (filled circles) and predicted (open circles) paired-comparison scale values for the two stimulus frequency conditions of Experiment 1.

those for Group B are constant or increase slightly, but those for Group L increase sharply. But between pairs of points associated with increase in loss frequency (3 to 5, or 1 to 3), the values for Group W increase substantially, those for Group B increase somewhat less, and those for Group L decrease.

Thus, the analysis in terms of the model confirms and extends the overall statistical comparisons of the three instruction groups. It seems apparent that subjects do indeed build up a memory structure that has some of the properties of a psychological scale and that serves to mediate highly efficient transfer performance. However, the scale cannot be identified with subjective probability in the traditional sense, for the values are directly related to objective probabilities only under very special circumstances. More generally, the scale values appear to reflect relative frequency information concerning categories of events that the individual encodes and rehearses under a given set of attentional and information-processing conditions.

There have, of course, been a number of studies by other investigators conducted under conditions in which predictive behavior could be expected to be based almost solely on categorical frequency information. Among the closest in design to the present series are those of Robbins and Medin (1971) and Allmeyer and Medin (1973). In those studies, training under an anticipation procedure was given concurrently on three pairs of alternatives and was followed by transfer tests. A novel feature was the use of equal π values for the members of each training pair (.8–.8, .5–.5, and .2–.2). Although the subjects' performance during training was not expected to, and did not, deviate significantly from the initial chance level of 50% on any of the pairs, the subsequent transfer tests showed that the subjects had, in fact, acquired information concerning the differing outcome frequencies.

In view of the task orientation (to choose the alternative "most likely to be correct"), we might expect that as in Experiment 1 of the present series, the parameter w in Equation 9 would be virtually equal to unity and transfer proportions would be predicted by Equation 6. However, there is a complication, in that although wins and losses were actually programmed independently for the two members of a training pair, the subjects probably tended to infer that when the alternative chosen on a trial proved incorrect, the other choice was the correct one. Thus, we need to introduce a parameter f, denoting the probability that subjects on a given condition actually make this inference.

In the study of Allmeyer and Medin (1973), instructions intended to further or to inhibit inferences about unchosen alternatives were given to two groups of subjects. The estimated values of f differ appropriately between the groups: .70 and .40, respectively. With ϕ set equal to 1 in Equation 6, since the data presented are presumably asymptotic, for the test pairs .8 versus .5, .8 versus .2, and .5 versus .2, we obtain predicted values of .52, .62, and .60, respectively, compared to observed values of .55, .60, and .56, respectively, for the first group. For the second group, the predicted values of .63, .71, and .72 correspond to observed values of .61, .74, and .64. In the study of Robbins and Medin (1971), instructions were nonspecific, so it is not surprising that the estimated value of f (.60) is between the estimated values of the other study; with this estimate, the asymptotic form of Equation 6 yields predicted values of .56, .64, and .58, in the same order, compared to observed values of .55, .60, and .56.

Another experimental design that provides an appropriate application of the model is probabilistic paired-associate learning. In a study reported by Voss, Thompson, and Keegan (1959), for example, one of two different response alternatives was assigned to each of nine stimuli. The items so constructed were presented in random order, the π values (ranging from .1 to .9) associated with an item determining which alternative would be presented as the correct response to the given stimulus on any trial. Estimated asymptotic choice proportions (representing averages of the observed proportions over the terminal trial blocks for three replications of Experiment 3) are plotted as a function

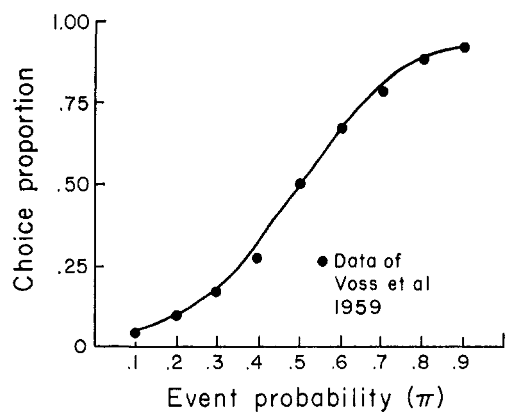


FIGURE 7. Estimated asymptotic choice proportions for the probabilistic paired-associate study of Voss, Thompson, and Keegan (1959) together with the predicted function from the relative frequency model (Equation 6).

of π value in Figure 7.[1] The orderly sigmoid function thus obtained seems quite well described by the values predicted from Equation 6 with ϕ set equal to .92. Once again, we find that in the absence of instructions to the contrary, subjects evidently store information only regarding winning ("correct") outcomes and scan the representations of these in memory when called upon to anticipate the outcomes.

Numerous other examples could be given, but those discussed above may suffice to show that the variation in particular formulas required to describe predictive behavior in different situations corresponds to the variation in modes of information processing and response selection induced by different tasks.

Application of the model to relative frequency judgments. Beyond providing a framework for the interpretation of various types of probability learning, the assumptions of the model bring out important similarities between the phenomena of probability learning and relative frequency judgments. In research designed to elucidate the relationships between recognition, verbal discrimination learning, and memory for frequency, a number of investigators have employed observation–test designs analogous to that of Experiment 1. However, these investigators have usually presented stimuli singly, rather than in pairs, on observation trials, and the subject's task on test trials has been to judge which member of the pair of test stimuli occurred more frequently during the observation series.

From the viewpoint developed here, the experiments of these investigators involve an especially simple case of the type of information processing assumed to underlie probability learning. Typically, rather long lists of items are used, and buffer items are inserted between the last occurrence of a to-be-tested item and the first test trial. Consequently, it is unlikely that episodic memory could play any appreciable role, and we must assume that the test responses are based on categorical memory for frequency information. To generate specific predictions from the present model, we need only treat each occurrence of an item as corresponding to a winning outcome in the probability learning situations. Since particular pairs of items usually are not tested repeatedly in these studies, or are retested only infrequently compared to probability learning studies, we should assume that the alternatives of a test pair are scanned in a random order. Thus, the appropriate function for purposes of predicting test performance is Equation 1, but with the parameter ϕ incorporated to allow for intermediate degrees of learning as in Equations 6 through 9; that is,

$$P_{ij} = (1 - \phi)(.5) + \phi \frac{W_i}{W_i + W_j}. \quad (10)$$

The principal implication of this analysis is that on tests of pairs of items presented at least once,[2] relative frequency judgments should tend to match the actual relative frequencies. This expectation seems well borne out in the data of a number of studies (Hintzman, 1969; Radtke, Jacoby, & Goedel, 1971; Underwood & Freund, 1970; Underwood, Zimmerman, & Freund, 1971). In Hintzman's data, for example, actual relative fre-

---

[1] I wish to thank James Voss for supplying the numerical data.

[2] Tests involving pairings of previously presented items with novel items bring in additional considerations, concerning stimulus generalization, which are beyond the scope of the present study.

quencies were

.60, .62, .67, .71, .80, .83, .86, .91,

and mean choice proportions,

.66, .66, .65, .75, .78, .89, .88, .91,

respectively. In the data of Radtke et al., relative frequencies were

.60, .67, .75, .80

and choice proportions,

.62, .66, .74, .80,

respectively. When the frequency of repetition of test pairs increases materially in this type of experiment, we should, of course, expect that subjects will begin to scan the test stimuli in the order of their relative frequencies and therefore that choice proportions will shift in the direction of the values predicted by Equation 6.

Probability estimates. A loose end that at present I can do little to tie into a formal theory concerns experiments in which subjects are called upon to produce numerical estimates of event probabilities. Typically, learning functions for mean probability estimates are very similar to those for proportions of predictive responses, and in the case of simple, noncontingent event schedules, tend to approach probability matching (Bauer, 1972; Neimark & Shuford, 1959).

I assume that estimates, like predictions, are based on categorical memory for frequency information, but the process by which an individual converts the memory representation into a numerical estimate of probability or frequency is something of a mystery. Perhaps the most parsimonious idea suggested by the present theoretical analysis is that the individual resamples the background contextual cues available on the test trial a number of times, generating a sequence of covert predictive responses, and then counts or "subitizes" these (as in reporting frequencies of dots from tachistoscopic displays) in order to generate a numerical response. On this assumption, the learning function for a noncontingent probability learning experiment would take the form of Equation 10, with P_{ij} denoting the subject's mean probability estimate with reference to event E_i, and with π_i (the probability of event E_i) taking the place of $W_i/W_i + W_j$. On the assumption that the parameter ϕ would in this case take the form $\phi = 1 - (1 - \theta)^n$, where n is the trial number and θ the stimulus–sampling fraction, this learning function would quite satisfactorily describe the obtained learning curves in the studies of Bauer (1972) and Neimark and Shuford (1959), with a bit of extra confirmation from the very similar pre- and postshift learning rates observed in the Bauer study.

Whatever the merits of this conjecture regarding the mechanism of response selection, considerable evidence seems to be accumulating to suggest that probability estimates, relative frequency judgments, and predictive behavior all share a common basis in associative memory.

REFERENCE NOTE

1. Brehmer, B. *Inductive inferences from uncertain information* (Umeå Psychological Report 78). University of Umeå, Sweden, 1974.

REFERENCES

Allmeyer, D. H., & Medin, D. L. Reward information and cue selection following multiple-cue probability learning. *Journal of Experimental Psychology*, 1973, *99*, 426–428.

Anderson, J. R., & Bower, G. H. *Human associative memory*. Washington, D.C.: V. H. Winston, 1973.

Atkinson, R. C., & Wickens, T. D. Human memory and the concept of reinforcement. In R. Glaser (Ed.), *The nature of reinforcement*. New York: Academic Press, 1971.

Bauer, M. Relations between prediction- and estimation-responses in cue-probability learning and transfer. *Scandinavian Journal of Psychology*, 1972, *13*, 198–207.

Binder, A., & Estes, W. K. Transfer of response in visual recognition situations as a function of frequency variables. *Psychological Monographs*, 1966, *80*(23, Whole No. 631).

Binder, A., & Feldman, S. E. The effects of experimentally controlled experience upon recognition responses. *Psychological Monographs*, 1960, *74*(9, Whole No. 496).

Björkman, M. Predictive behavior. Some aspects based on an ecological orientation. *Scandinavian Journal of Psychology*, 1966, *7*, 43–57.

Bower, G. H. Choice point behavior. In R. R. Bush & W. K. Estes (Eds.), *Studies in mathematical learning theory*. Stanford, Calif.: Stanford University Press, 1959.

Bush, R. R., & Mosteller, F. *Stochastic models for learning*. New York: Wiley, 1955.

Castellan, N. J., Jr., & Edgell, S. E. An hypothesis generation model for judgment in nonmetric multiple-cue probability learning. *Journal of Mathematical Psychology*, 1973, *10*, 204–222.

Cohen, J. *Behavior in uncertainty*. New York: Basic Books, 1964.

Ekstrand, B. R., Wallace, W. P., & Underwood, B. J. A frequency theory of verbal discrimination learning. *Psychological Review,* 1966, *73,* 566–578.

Estes, W. K. Component and pattern models with Markovian interpretations. In R. R. Bush & W. K. Estes (Eds.), *Studies in mathematical learning theory.* Stanford, Calif.: Stanford University Press, 1959.

Estes, W. K. A random-walk model for choice behavior. In K. J. Arrow, S. Karlin, & P. Suppes (Eds.), *Mathematical methods in the social sciences, 1959.* Stanford, Calif.: Stanford University Press, 1960.

Estes, W. K. Theoretical treatments of differential reward in multiple-choice learning and two-person interactions. In J. H. Criswell, H. Solomon, & P. Suppes (Eds.), *Mathematical methods in small group processes.* Stanford, Calif.: Stanford University Press, 1962.

Estes, W. K. All-or-none processes in learning and retention. *American Psychologist,* 1964, *19,* 16–25.

Estes, W. K. Transfer of verbal discriminations based on differential reward magnitudes. *Journal of Experimental Psychology,* 1966, *72,* 276–283.

Estes, W. K. Reinforcement in human learning. In J. Tapp (Ed.), *Reinforcement and behavior.* New York: Academic Press, 1969.

Estes, W. K. Elements and patterns in diagnostic discrimination learning. *Transactions of the New York Academy of Sciences,* 1972, *34,* 84–95. (a)

Estes, W. K. Reinforcement in human behavior. *American Scientist,* 1972, *60,* 723–729. (b)

Estes, W. K. Research and theory on the learning of probabilities. *Journal of the American Statistical Association,* 1972, *67,* 81–102. (c)

Estes, W. K. Memory and conditioning. In F. J. McGuigan & D. B. Lumsden (Eds.), *Contemporary approaches to conditioning and learning.* Washington, D.C.: V. H. Winston, 1973.

Estes, W. K. Some functions of memory in probability learning and choice behavior. In G. H. Bower (Ed.), *The psychology of learning and motivation* (Vol. 10). New York: Academic Press, in press. (a)

Estes, W. K. Structural aspects of associative models for memory. In C. N. Cofer (Ed.), *The structure of human memory.* San Francisco: Freeman, in press. (b)

Estes, W. K., & Straughan, J. H. Analysis of a verbal conditioning situation in terms of statistical learning theory. *Journal of Experimental Psychology,* 1954, *47,* 225–234.

Feigenbaum, E. A. Simulation of verbal learning behavior. In E. A. Feigenbaum & J. Feldman (Eds.), *Computers and thought.* New York: McGraw-Hill, 1963.

Feldman, H., & Hanna, J. F. The structure of responses to a sequence of binary events. *Journal of Mathematical Psychology,* 1966, *3,* 371–387.

Friedman, M. P. et al. Two-choice behavior under extended training with shifting probabilities of reinforcement. In R. C. Atkinson (Ed.), *Studies in mathematical psychology.* Stanford, Calif.: Stanford University Press, 1964.

Friedman, M. P., Rollins, H., & Padilla, G. The role of cue validity in stimulus compounding. *Journal of Mathematical Psychology,* 1968, *5,* 300–310.

Goldberg, L. R. Man versus model of man: A rationale, plus some evidence, for a method of improving clinical inferences. *Psychological Bulletin,* 1970, *73,* 422–432.

Greeno, J. G. *Elementary theoretical psychology.* New York: Addison-Wesley, 1968.

Hintzman, D. L. Apparent frequency as a function of frequency and the spacing of repetitions. *Journal of Experimental Psychology,* 1969, *80,* 139–145.

Jenkins, H. M., & Ward, C. W. Judgment of contingency between responses and outcomes. *Psychological Monographs,* 1965, *79*(1, Whole No. 594).

Johnson, N. F. The role of chunking and organization in the process of recall. In G. H. Bower (Ed.), *The psychology of learning and motivation* (Vol. 4). New York: Academic Press, 1970.

Kahneman, D., & Tversky, A. Subjective probability: A judgment of representativeness. *Cognitive Psychology,* 1972, *3,* 430–454.

Luce, R. D. *Individual choice behavior. A theoretical analysis.* New York: Wiley, 1959.

Luce, R. D., & Suppes, P. Preference, utility, and subjective probability. In R. D. Luce, R. R. Bush, & E. Galanter (Eds.), *Handbook of mathematical psychology* (Vol. 3). New York: Wiley, 1965.

Mandler, G., Cowan, P. A., & Gold, C. Concept learning and probability matching. *Journal of Experimental Psychology,* 1964, *67,* 514–522.

Myers, J. L. Probability learning and sequence learning. In W. K. Estes (Ed.), *Handbook of learning and cognitive processes* (Vol. 3). Hillsdale, N.J.: Lawrence Erlbaum Associates, in press.

Neimark, E. D. *Effects of type of non-reinforcement and number of alternative responses in two verbal conditioning situations.* Unpublished doctoral dissertation, Indiana University, 1953.

Neimark, E. D., & Shuford, E. H. Comparison of predictions and estimates in a probability learning situation. *Journal of Experimental Psychology,* 1959, *57,* 294–298.

Radtke, R. C., Jacoby, L. L., & Goedel, G. D. Frequency discrimination as a function of frequency of repetition and trials. *Journal of Experimental Psychology,* 1971, *89,* 78–84.

Restle, F. *Psychology of judgment and choice: A theoretical essay.* New York: Wiley, 1961.

Restle, F. The selection of strategies in cue learning. *Psychological Review,* 1962, *69,* 329–343.

Restle, F. Significance of all-or-none learning. *Psychological Bulletin,* 1965, *64,* 313–325.

Restle, F., & Brown, E. R. Serial pattern learning. *Journal of Experimental Psychology,* 1970, *83,* 120–125.

Robbins, D., & Medin, D. L. Cue selection after multiple-cue probability training. *Journal of Experimental Psychology,* 1971, *91,* 333–335.

Shuford, E. H., Jr. Some Bayesian learning processes. In M. W. Shelly II, & G. L. Bryan (Eds.), *Human judgments and optimality.* New York: Wiley, 1964.

Smedslund, J. The concept of correlation in adults. *Scandinavian Journal of Psychology,* 1963, *4,* 165–173.

Suppes, P., & Atkinson, R. C. *Markov learning models for multi-person interactions.* Stanford, Calif.: Stanford University Press, 1960.

Tulving, E. Theoretical issues in free recall. In T. R. Dixon & D. L. Horton (Eds.), *Verbal behavior and general behavior theory.* Englewood Cliffs, N.J.: Prentice-Hall, 1968.

Tulving, E. Episodic and semantic memory. In E. Tulving & W. Donaldson (Eds.), *Organization of memory.* New York: Academic Press, 1972.

Underwood, B. J. Recognition memory. In H. H. Kendler & J. T. Spence (Eds.), *Essays in neobehaviorism.* New York: Appleton-Century-Crofts, 1971.

Underwood, B. J., & Freund, J. S. Relative frequency judgments and verbal discrimination learning. *Journal of Experimental Psychology,* 1970, *83,* 279–285.

Underwood, B. J., Zimmerman, J., & Freund, J. S. Retention of frequency information with observations on recognition and recall. *Journal of Experimental Psychology,* 1971, *87,* 149–162.

Voss, J. F., Thompson, C. P., & Keegan, J. H. Acquisition of probabilistic paired associates as a function of $S\text{-}R_1$, $S\text{-}R_2$ probability. *Journal of Experimental Psychology,* 1959, *58,* 390–399.

Weir, M. W. Developmental changes in problem solving strategies. *Psychological Review,* 1964, *71,* 473–490.

Wolin, B. R., Weichel, R., Terebinski, S. J., & Hansford, E. A. Performance on complexly patterned binary event sequences. *Psychological Monographs,* 1965, *79*(7, Whole No. 600).

Yellott, J. I. Probability learning with noncontingent success. *Journal of Mathematical Psychology,* 1969, *6,* 541–575.

(Received November 13, 1974)